Optimal Allocation of Electromagnetic Spectrum to Support Tactical Wideband Communications

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ABSTRACT

The Marine Corps is currently developing, purchasing, and fielding tactical wideband radios capable of connecting highly mobile units operating in rugged terrain over long distances. Wideband radios offer tremendous new capabilities, including high data rates and automatic traffic relay, but they have large electromagnetic spectrum requirements. We explore the challenges faced by a spectrum manager in allocating the minimum number of channels to support wideband communications for Marine Corps forces conducting tactical operations. A spectrum manager must consider not only the scarcity of available spectrum, but the technological limitations of the radios being supported. Specifically, the performance of a wideband radio depends greatly on transmission power, antenna gain, and other technical constraints, and on the amount of co-channel interference it receives from other radios operating on the same channel.

We create a network communications model to capture the most relevant aspects of wideband communications, and use high-fidelity tools to simulate radio propagation in three realistic combat scenarios. We then formulate an interference-aware minimum-order channel assignment problem as a pure 0-1 integer program to determine the minimum number of channels required to support wideband communications. We examine an exact method of solving the problem, and develop and test a heuristic algorithm that in practice provides solutions at least as good as the exact method when processing time is not unlimited. We find that in the largest scenario, there is insufficient spectrum available to support the full wideband spectrum requirement, even with optimized channel reuse. To our knowledge, we are the first to describe an algorithm for solving an interference-aware minimum-order channel assignment problem for tactical wideband radio communications, and are the first to rigorously examine and quantify the ability to reuse wideband radio channels within the Marine Corps.

INTRODUCTION

Description of Problem

Tactical forces within the US Marine Corps (USMC) are becoming increasingly dependent on the rapid, reliable transfer of information throughout the battlespace. *Marine Corps Vision and Strategy 2025* (USMC, 2008a) states, ''the Marine Corps will integrate C2 [command and control] and ISR [intelligence, surveillance, and reconnaissance] capabilities down to the squad level,'' and, ''we will aggressively pursue integrated microtechnologies, such as a secure communications personal data apparatus that communicates via the spoken word, data, and imagery.'' The Marine Corps document *A Concept for Enhanced Company Operations* (2008b) emphasizes ''support to highly mobile forces with on-the-move/over-the-horizon communications for disparate tactical nodes,'' and states that to achieve this, ''tactical units must gravitate from push-to-talk radio systems to mobile ad-hoc mesh networking.''

The Marine Corps is currently developing, purchasing, and fielding tactical wideband radios capable of connecting highly mobile units operating in rugged terrain over long distances with relatively low-power radios (Goulding, 2009). The wideband radios we consider connect wirelessly to each other to form a mobile ad hoc network (MANET), an autonomous communications system where each wideband radio serves as a mobile node. These radios may move and connect in wireless, dynamic, multihop topologies, and exhibit self-learning, self-healing behavior (Corson and Macker, 1999; Aggelou, 2005), i.e., individual radios may automatically connect and disconnect from a MANET without any user interaction. A MANET system comprises physical radios and the associated networking protocols, waveforms, and modulation schemes. Each wideband radio in a MANET is a terminal device for voice or digital communications, and may concurrently serve as a relay device for other radios in the network. In this way, MANETs are similar to client-mesh *wireless mesh networks* (*WMNs*) (Zhang et al., 2006, pp. 564–567), where client devices perform routing functions.

Wideband radios offer tremendous new capabilities, including high data rates and automatic traffic relay, but have large electromagnetic spectrum requirements. Technological limits constrain the number of wideband radios that can be assigned to the same channel (i.e., a contiguous portion of spectrum), and the channels used by

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OR METHODS: Integer programming; Network methods; Simulation

wideband radios are larger than that used by legacy narrowband radios. For instance, 1.2 megahertz (MHz) wideband channels occupy 48 times more spectrum than 25 kilohertz (kHz) narrowband channels used for voice-only communications. The introduction of this new wideband capability will challenge the status quo for spectrum allocation, and future communications planning must balance the requirements of new wideband networks and greater data capabilities with less capable, legacy narrowband networks that require less spectrum.

Meanwhile, the Marine Corps will continue to operate in environments with increasing restrictions on spectrum use, both in the US and abroad. Wireless communications traffic from civilian, joint, and coalition networks will increasingly clutter the electromagnetic (EM) spectrum. Efficient allocation of available spectrum is required to ensure that Marine Air-Ground Task Forces (MAGTFs) are able to fully utilize new tactical wideband radio assets.

We explore the challenges faced by a spectrum manager in allocating available channels to support wideband communications for Marine Corps forces conducting tactical operations. A spectrum manager must consider not only the scarcity of available spectrum, but the technological limitations of the radios being supported. Specifically, the performance of a wideband radio (e.g., the data rate) depends greatly on the amount of interference it receives. The interference can be naturally occurring (such as solar radiation), intentional (such as jamming), or unintentional (such as from other nearby radios). To minimize the number of channels required, a spectrum manager must efficiently reuse available channels while being mindful of interference and other technological constraints, such as transmission power and antenna gain. Current automated methods of channel allocation, including Spectrum XXI (Defense Information Systems Agency, 2013), do not consider interference among a large number of mobile transmitters over multiple time periods, nor do they provide a rigorous method for minimizing the number of required channels.

We first create a network communications model to capture the most relevant aspects of wideband communications. Based on force structure and radio fielding plans, we then calculate the unconstrained channel requirements to support wideband communications for MAGTFs in three realistic combat simulations. We then formulate an interference-aware minimum-order channel assignment problem to determine the minimum number of channels required to support wideband communications, given optimal channel reuse and considering the effects of co-channel interference and the technical specifications of the radios. We show the size of this problem grows exponentially in the number of radios requiring channels, and it is hence infeasible for spectrum managers to solve anything larger than trivially small problems without automation. We examine the use of the branch-and-cut method to exactly solve the problem, and develop and test a heuristic algorithm which in practice provides answers at least as good as branch-and-cut when processing time is limited. Next, we use subject matter expertise to determine realistic channel allocations for each scenario. A positive difference between this allocation and the solution to our channel assignment problem represents spectrum shortfall, a measure of the inability to operate wideband radios at full capability. We outline the implications of these shortfalls on the use of wideband radios.

Much of the research presented in this paper was conducted as part of the MAGTF Wideband Spectrum Requirement and Allocation Study, executed by the Marine Corps Combat Development Command (MCCDC) and sponsored by the Marine Corps Command and Control Integration Division (C2ID). This work has been used by the study sponsor to quantify spectrum requirements and inform acquisition decision-making. See MCCDC (2013) for the complete study report.

Previous Work

Research into the problem of allocating spectrum has been growing rapidly with the spread of wireless telephony (including both voice and data networks) and satellite communications (Aardal et al., 2007). Hale (1980) wrote what has been described as a landmark paper in the field of frequency assignment problems (Murphey et al., 1999). He differentiates the *frequency assignment* problem (where assigned frequencies may be noncontiguous) from the *channel assignment* problem (where assigned frequencies are in a contiguous block) that we consider. He also recognizes two possible figures of merit for this family of problems: *span* (the total range of frequencies assigned) and *order* (the total number of channels), which we consider.

Murphey et al. (1999) observe that though there is extensive research into the channel assignment problem, it remains a notoriously difficult problem to solve. Metzger (1970) is generally credited with first observing the possibility of using optimization techniques for solving these problems. He relates the class of problems to the vertex or graph coloring problem (Gould, 1998). Figure 1 provides a visualization of a graph coloring problem, where the color of each shape represents a channel assignment. In the general graph coloring problem, the only constraint is that no two adjacent shapes have the same assignment (i.e., are the same color). This constraint and the problem itself seem simple, yet the problem is NP-complete (Skiena, 1990, pp. 211–212; Cuppini, 1994).

Note the interference-aware channel assignment problem we consider is considerably more complex than the graph coloring problem depicted in Figure 1. Rather than assigning channels to discrete geographic areas, we must assign channels to military units consisting of many radios, each a source of electromagnetic radiation. We also consider the effects of terrain and EM propagation, so in practice coverage areas are not abutting polygons but discontinuous and overlapping ''splotches.'' The ability to reuse channels (and thus minimize the number required) is limited not by physical proximity, but by the sum total of co-channel interference from all other radios assigned to different units.

Murphey et al. (1999) observe that due to the complexity of the problem, real-world practitioners often rely on *sequential methods* that assign a channel to each radio or network one at a time. They contrast these methods to the

Figure 1. Example of the graph coloring problem. Our problem is considerably more complex, as we calculate coverage areas based on electromagnetic propagation over terrain. Underlying image courtesy of Google Maps (http://maps.google.com).

exact methods based on the graph coloring problem. Metzger (1970) describes several heuristic methods that make assignments sequentially. An *exhaustive* method attempts to assign the lowest available channel. A *uniform* method attempts to use that channel that has been used the least. A *requirement exhaustive* method attempts to use each channel in order. We explore the use of an exact method, and find that the complexity of our formulation and the size of our data sets oblige us to rely on heuristic methods for our larger scenarios.

Aardal et al. (2007) conduct a far-reaching survey of contemporary research into solution techniques for channel assignment problems. They differentiate *dynamic channel assignment* problems (where channel assignments may vary over time) from the *fixed channel assignment* problem we consider. We roughly follow Aardal et al. in formulating our interference-aware minimum-order problem, though our problem considers the additional complexity of the operation of a MANET.

Wu et al. (2010) create a channel assignment algorithm for multichannel wireless mesh networks. Their objective is to maximize throughput, and they assign a fixed interference range to each radio, whereas our objective is to minimize the number of channels assigned, and we calculate interference using high-fidelity simulation for each pair of radios. Kyasanur and Vaidya (2006) consider channel assignment for MANETs, but they purposely use all available channels. Voudouris and Tsang (1998) use a guided local search heuristic to minimize the use of channels, but use simple interference constraints that do not consider MANET-capable radios. The European Cooperation on the Long Term in Defense (EUCLID) Combinatorial Algorithms for Military Applications (CALMA) project (Tiourine et al., 1995; Aardal et al., 2002, 1996) uses a branch-and-cut algorithm to minimize the number of channels in a channel assignment problem. We also explore the use of the branch-and-cut algorithm, but our problem formulation differs in that we consider additive co-channel interference constraints (e.g., Katzela and Naghshineh, 1996), rather than the pairwise interference constraints that are prevalent in the literature. We find that due to this additional complexity, we cannot use branch-and-cut to find exact solutions for larger problems in a reasonable amount of time.

In previous work, the author considers the use of different MANET architectures when sufficient spectrum is assumed to be available (MCCDC, 2011; Nicholas et al., 2013). This research complements and extends that work by fixing the MANET architecture and determining allocation when spectrum is a scarce resource. To our knowledge, we are the first to describe an algorithm for solving an interference-aware minimum-order channel assignment problem for tactical wideband radio communications. We are also the first to rigorously examine and quantify the ability to reuse wideband channels within the Marine Corps.

This paper is organized as follows. In the next section, we describe in detail our techniques for modeling tactical communications networks and the methods used to allocate electromagnetic spectrum. We then describe several analyses using these models and methods, and briefly summarize our findings. We conclude with suggestions for follow-on research.

INTERFERENCE-AWARE CHANNEL ASSIGNMENT PROBLEM

Communications Network Topology

We create a network model to simulate key aspects of a MANET formed by tactical wideband radios. Let $r \in R$ (alias s) represent each wideband radio. Each radio is permanently assigned to a logical unit $u \in U$, indicated by the set of logical arcs $(r, u) \in L$. A unit may represent a tactical organization such as an infantry company or reconnaissance team. Let the set of nodes N (indexed by n) consist of both radios R and units U, i.e., $n \in N = R \cup U$.

Let a channel $c \in C$ be a contiguous range of EM frequencies, where C is the set of available orthogonal (i.e., noninterfering) channels. Each unit u and the radios $r \in R$ assigned to it require a channel assignment. Let the binary variable X_n^c indicate whether node n (either a radio or a unit) is using channel c:

$$X_n^c = \begin{cases} 1 & \text{if node } n \text{ (i.e., radio or unit) uses channel } c \\ 0 & \text{otherwise} \end{cases} \quad \forall n \in N, c \in C.$$

Each radio is assigned the same channel as its associated unit, so

$$X_r^c = X_u^c \quad \forall c \in C, (r,u) \in L.$$

To ensure each unit u is assigned one and only one channel, the problem contains the constraint:

$$\sum_{c \in C} X_u^c = 1 \quad \forall u \in U.$$

Let the binary variable Y^c indicate whether channel c is being used:

$$Y^c = \begin{cases} 1 & \text{if channel } c \text{ is used} \\ 0 & \text{otherwise} \end{cases} \quad \forall c \in C.$$

Since the goal is to minimize the total number of channels required, our objective function is:

$$\min \sum_{c \in C} Y^c.$$

Let $(r, s) \in W$ indicate the set of all wireless arcs between all radios $r, s \in R$. These arcs represent both intentional EM transmissions between radios assigned to the same unit, and unwanted interference from all other radios assigned to the same channel $c \in C$. These arcs exist in both directions, and each radio can receive transmissions from any other radio, so $|W| = |R|(|R| - 1)$.

A unit $u \in U$ forms a MANET among its assigned radios using the available wireless arcs $(r, s) \in W$: $(r, u) \in L$, $(s, u) \in L$. Each MANET enables the exchange of communications traffic between all radios and a network control radio, such as the infantry company commander or reconnaissance team leader. This bidirectional connectivity to a single radio ensures that radios within each unit are strongly connected (i.e., a directed path exists between each pair of radios) (Ahuja et al., 1993, p. 27). Technological limits of the radios constrain the number of radios that can be assigned to the same unit; we assume a limit of 30 radios.

Figure 2 shows two separate units (indicated in blue and green) and their assigned radios. The solid lines indicate bidirectional wireless arcs $(r, s) \in W$ between radios. Any radio (e.g., radio r in Figure 2) communicates with its network control radio (e.g., radio s) via these arcs (a radio may route through other radios in the same unit to reach the network control radio). All radios are subject to co-channel interference from any other radios assigned to different units but operating on the same channel, indicated by dashed gray arrows directed to r (other lines withheld for clarity).

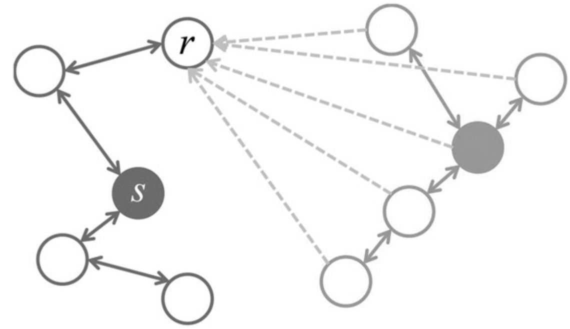

Figure 2. Simple example of two units (indicated in blue and green) with network control radios (solid circles) and other radios (open circles). Wireless arcs are indicated by arrows. The radios within each unit must be capable of bi-directional communication with their unit's network control radio via direct communication or routing through other radios in the same network. All radios are subject to co-channel interference (dashed arrows) from other radios assigned to different units but operating on the same channel.

We do not model connections between units; that is, radios may communicate only within their own units. In practice, connectivity between units is provided by satellite, fiber optic cable, or other backhaul network.

Calculating Received Signal Strength

To calculate both co-channel interference and the strength of desired wireless transmissions between intra-unit radios, we calculate the received signal strength (RSS) ρ_{rs} along all wireless arcs $(r, s) \in W$ in dBm using the standard link budget formula (Olexa, 2005, p. 79):

$$\rho_{rs} = power_r + g_r - l_r - l_{path} - l_{misc} + g_s - l_s \quad \forall (r,s) \in W$$

where $power_r$ is transmitted power in dBm, g_r and g_s are respectively the gains of the radios

r and s in dB, l_r and l_s are respectively the losses (i.e., from cables, connectors, etc.) of the radios in dB, l_{path} is total path loss in dB, and l_{misc} is miscellaneous loss or fade margin in dB. All of the terms are input data, determined by the equipment and environment, except for the total path loss l_{path}, which depends on the physical position of radios r and s and the intervening terrain.

Our formulation allows the use of any method for computing l_{path}, including the irregular terrain model (ITM) (Longley and Rice, 1968) and Hata-COST 231 (COST, 1999). We instantiate our scenarios in Systems Toolkit (STK) (Analytical Graphics, Inc., 2013) and then use the Terrain Integrated Rough Earth Model (TIREM) of Alion Science & Technology Corporation (Alion, 2007) to calculate l_{path}. STK allows us to consider the movement of various types of platforms (e.g., ground troops, vehicles, aircraft, etc.) in a three-dimensional environment, and TIREM samples terrain elevation to compute path loss, considering the effects of free space loss, diffraction around obstacles, and atmospheric absorption and reflection.

Although TIREM is computationally more expensive than simpler models, it provides fairly accurate results. For line-of-sight propagation in commonly used frequency ranges, Eppink and Kuebler (1994) compare TIREM predictions and actual measurements. They find a difference with a mean of −2.8 dB and a standard deviation of 8.9 dB, which is very accurate considering the speed and relative simplicity of the model.

Calculating Connectivity and Interference

To calculate the strength of connectivity between each radio and its network control radio, we use Dijkstra's algorithm (Dijkstra, 1959) to calculate the shortest path from each radio to its assigned network control radio. Arc cost is defined to be inversely proportional to the RSS ρ_{rs} between radios. This methodology favors paths that have both fewer links and higher received signal strengths. We assume a radio will be disconnected from its assigned network control radio if it is unable to communicate along this shortest path, as all other paths will be more costly (i.e., consist of more links and/or links of weaker signal strength). Along each path and at each radio $s \in R$, we follow Aardal et al. (2007) and precalculate the maximum allowable interference in watts $max\_interference^c_s$. This calculation is based on the RSS ρ_{rs} between radios and each particular radio's required signal-to-interference ratio (SIR), a measure of signal quality (Poisel, 2011, p. 319). Any co-channel interference above this level severs the shortest path and thus disconnects the radio from its assigned network control radio.

Among radios not assigned to the same unit but operating on the same channel, the RSS ρ_{rs} represents co-channel interference. The magnitude of co-channel interference along all arcs $(r, s) \in W$ for each available channel $c \in C$ is precalculated in watts, and is indicated by $interference^c_{rs}$. (We simulate transmissions between all radios, though in practice some arcs may represent negligible or zero interference.) Note that radios may only be subject to interference if they are both assigned to the same channel, so one possible constraint is:

$$interference^c_{rs} X^c_r X^c_s \leq max\_interference^c_s \quad \forall (r,s) \in W, c \in C.$$

That is, a radio $s \in R$ may be assigned a particular channel $c \in C$ only if the interference from any other single radio is at or below the precalculated $max\_interference^c_s$ threshold. Following Katzela and Naghshineh (1996) and Ståhlberg (2000), we assume the cumulative effects of jamming sources on the same channel are additive (in watts) at each receiver. That is, a radio $s \in R$ may be unable to use a channel $c \in C$ because the total sum of interference exceeds the threshold $max\_interference^c_s$, even if the interference received from any single radio is less than the threshold. Summing along all arcs yields:

$$\sum_{r:(r,s)\in W} interference^c_{rs} X^c_r X^c_s \leq max\_interference^c_s \quad \forall s \in R, c \in C.$$

To linearize this constraint, we introduce a constant *bound* as an upper bound on the possible interference at any radio $r \in R$ or channel $c \in C$. This constant, specified in watts, need not be calculated precisely. We thus obtain:

$$\sum_{r:(r,s)\in W} interference_{rs}^{c} X_r^c \le max\_interference_s^c + bound\left(1 - X_s^c\right) \quad \forall s \in R, c \in C.$$

Our minimum-order (MO) channel assignment problem (CAP) formulation is summarized in the textbox below.

The minimum-order channel assignment problem (MO-CAP) is a pure 0-1 integer program. The objective function (M0) minimizes the sum of assigned channels. Constraints (M1) ensure that each channel utilized by a unit $u \in U$ is counted toward the objective function. Constraints (M2) require the assignment of one channel to each unit. Constraints (M3) require that each radio uses the same channel as its assigned unit. Constraints (M4) ensure that the sum total of co-channel interference at each radio is below the maximum threshold.

MO-CAP is NP-complete (Skiena, 1990, pp. 211–212; Cuppini, 1994); the time required to find solutions to large problems grows very quickly (Harary, 1994, p. 127). When channels are indistinguishable, the number of possible solutions to the problem can be calculated as a Bell number B_n, i.e., the number of ways a set of n elements can be partitioned into nonempty subsets (Bell, 1934). In this case, the goal is to partition $|U|$ units into

Formulation MO-CAP

Index Use

$n \in N$	node (either radio or unit)
$r \in R \subset N$	radio (alias s)
$u \in U \subset N$	unit
$c \in C$	channel
$(r,u) \in L$	logical arc between radio $r \in R$ and unit $u \in U$
$(r,s) \in W$	wireless arc between radios r and $s \in R$

Input Data

$interference_{rs}^{c}$	interference on $c \in C$ along wireless arc $(r,s) \in W$	[watts]
$max\_interference_s^c$	max allowable interference on $s \in R$ and $c \in C$	[watts]
$bound$	upper bound on possible interference	[watts]

Decision Variables

X_n^c	binary variable indicating if node n (radio or unit) is using channel c	[none]
Y^c	binary variable indicating if channel c is being used	[none]

Formulation

$$\min_{X,Y} \quad \sum_{c\in C} Y^c \qquad \text{(M0)}$$

$$s.t. \quad X_u^c \le Y^c \qquad \forall u \in U, c \in C \qquad \text{(M1)}$$

$$\sum_{c\in C} X_u^c = 1 \qquad \forall u \in U \qquad \text{(M2)}$$

$$X_r^c = X_u^c \qquad \forall c \in C, (r,u) \in L \qquad \text{(M3)}$$

$$\sum_{r:(r,s)\in W} interference_{rs}^{c} X_r^c \le max\_interference_s^c + bound\left(1 - X_s^c\right) \qquad \forall s \in R, c \in C \qquad \text{(M4)}$$

$$X_n^c \in \{0,1\} \qquad \forall n \in N, c \in C \qquad \text{(M5)}$$

$$Y^c \in \{0,1\} \qquad \forall c \in C \qquad \text{(M6)}$$

sets of channels. Bell numbers grow exponentially, and can be generated using the sum and recurrence relation:

$$B_n = \sum_{k=0}^{n-1} B_k \binom{n-1}{k}$$

or using Dobiński's formula (Dobiński, 1877):

$$B_n = \frac{1}{e}\sum_{k=0}^{\infty}\frac{k^n}{k!}.$$

When channels are distinguishable (e.g., occupying different frequency bands) and a total of $|C|$ channels are available, the number of solutions is $|C|^{|U|}$. In either case, the number of solutions quickly grows very large. For example, with distinguishable channels and $|C| = |U|$ (to ensure a feasible solution) there are 46,656 solutions to the Marine expeditionary unit (MEU) scenario, 1.33×10^{33} solutions to the Marine expeditionary brigade (MEB) scenario, and 3.03×10^{244} solutions to the Marine expeditionary force (MEF) scenario. With such a vast number of possible solutions, it is extremely unlikely spectrum managers can find the optimal solution to large problems. Clearly, automation is required to find efficient channel assignments for anything other than trivial problems.

SOLUTION METHODS

Exact Solution Method

A combinatorial optimization technique known as branch-and-bound is particularly efficient at solving pure 0-1 (or *binary*) integer problems (Nemhauser and Wolsey, 1999, pp. 456–457). We formulate our problem to take advantage of this. We use the General Algebraic Modeling System (GAMS) (GAMS Development Corporation, 2013) and CPLEX optimizer (IBM, 2013a) to run branch-and-cut, a generalization of the branch-and-bound method (Chen et al., 2010, pp. 305–306). We use Dell Precision T5500 desktop computers with twelve 3.47 gigahertz (GHz) Xeon processors and 72 gigabytes (GB) of random access memory (RAM).

CPLEX finds optimal solutions to the MEU scenario (comprising up to 131 radios) very quickly. With the MEB scenario (comprising 641 radios), CPLEX is able to find an optimal solution only occasionally, at certain time steps and SIR threshold levels. On other occasions, it finds only a feasible solution during 24 hours of processing. The solver fails to find any feasible solution to the MEF scenario (comprising 1887 radios) in more than two weeks of processing. This failure to converge to an optimal solution is due not only to the large number of variables in this scenario, but because CPLEX is sensitive to the vast range of input values (IBM, 2013b). Specifically, the *interference* and *max_interference* values in the cumulative interference constraints (M4) can vary by 24 orders of magnitude. The following CPLEX runtimes are recorded:

- MEU: $<$ 2 seconds (optimal)
- MEB: 5 seconds (optimal) to 24 hours (suboptimal)
- MEF: $>$ 2 weeks (no solution found)

Heuristic Solution Method

In order to quickly find a feasible solution to larger instances of the problem, we create and implement a greedy algorithm (Winston, 2004, pp. 457) that calculates a locally optimal solution in a finite number of steps. The algorithm first pre-processes the input data, summing (by unit) the total interference received at each radio from all other radios not assigned to the same unit. The algorithm then determines if there are any units that cannot (due to co-channel interference) share a channel with another unit; if so, such units are assigned their own channels. This is similar to Box's (1978) method of assigning channels to the most difficult radios first.

The algorithm then enters the main processing loop, essentially "packing" units onto a channel until no additional units can be assigned with acceptable co-channel interference. We keep track of this using the variable $interferenceMargin_r^c$, indicating the remaining amount of co-channel interference each radio r can suffer before being unable to operate. When a channel is "full," a new channel is selected and the process continues until no unassigned units are remaining.

Our method has two primary parameters: the method of selecting the next available channel, and the method of selecting the next unit to assign to the working channel. During empirical testing we find that the quality of solutions is fairly robust to the choice of channel, i.e., in these particular scenarios there is no clear advantage to the use of higher or lower frequency channels. When assigning units to a working channel, we choose that unassigned unit that least interferes with that already assigned unit on the working channel that is closest to suffering unacceptable co-channel interference.

The following pseudocode describes our heuristic algorithm *PackChannels*:

The function CalculateEligibleUnits supports the algorithm *PackChannels* by determining the units that are eligible to be assigned to the given channel, considering interference constraints:

We implement our heuristic in Microsoft Visual Basic for Applications (VBA) (Microsoft, 2013). Although this constructive heuristic cannot provide a certificate of optimality for any particular solution, it is guaranteed to find a feasible, locally optimal solution in a finite number of steps. In practice we find

Algorithm *PackChannels*

Input: Number of units requiring channels $|U|$; $max\_interference_s^c \; \forall s \in R, c \in C$
Output: $X_u^c \; \forall u \in U, c \in C$; $Y^c \; \forall c \in C$

begin
  $currentChannel \leftarrow 0$
  $numberAssignedUnits \leftarrow 0$
  $interferenceMargin_r^c \leftarrow max\_interference_s^c$
  for $i = 1$ to $|U|$
    Assign individual channel to any unit that cannot share channels
    $numberAssignedUnits \leftarrow numberAssignedUnits + 1$
  next;
  while $(numberAssignedUnits < |U|)$ **do**
    $currentChannel \leftarrow$ next available channel
    if $(|U| - numberAssignedUnits > 2)$
      $nextUnit \leftarrow$ the unassigned unit that receives the least interference from all other unassigned units
    else
      $nextUnit \leftarrow$ the first remaining unit
    endif;
    $X_{nextUnit}^{currentChannel} \leftarrow 1$
    $eligibleUnits \leftarrow$ **CalculateEligibleUnits** $(currentChannel)$
    while $(|eligibleUnits| > 0)$ **do**
      $weakestUnit \leftarrow$ the unit already assigned to $currentChannel$ with smallest remaining $interferenceMargin_r^{currentChannel}$
      $leastInterferer \leftarrow$ the $eligibleUnit$ that least interferes with $weakestUnit$
      $eligibleUnits \leftarrow eligibleUnits \backslash leastInterferer$
      $X_{leastInterferer}^{currentChannel} \leftarrow 1$
      $numberAssignedUnits \leftarrow numberAssignedUnits + 1$
      Update $interferenceMargin_r^{currentChannel}$ for all radios assigned to $currentChannel$
      $eligibleUnits \leftarrow$ **CalculateEligibleUnits**$(currentChannel)$
    end;
    $Y^{currentChannel} \leftarrow 1$
  end;
end;

Function ***CalculateEligibleUnits***$(givenChannel)$

```
// Calculate and return eligibleUnits (the set of units eligible for assignment) for the
givenChannel
begin
    eligibleUnits = {}
    for each unassigned unit u
        if u can be assigned to givenChannel and not cause unacceptable co-channel
        interference
        eligibleUnits ← eligibleUnits ∪ u
        end;
    end;
return eligibleUnits;
```

it can provide good solutions in seconds, and occasionally finds better solutions than the feasible solutions found using CPLEX (i.e., when CPLEX is not run to optimality due to time constraints):

- MEU: <0.5 seconds (at optimality)
- MEB: 15 seconds (occasionally better than feasible solution found using CPLEX)
- MEF: $<$ 7 minutes (no solution found using CPLEX)

In Figure 3 we compare the solution values of MO-CAP obtained using our heuristic to those obtained using CPLEX. A point below the diagonal axis indicates that the heuristic finds a better solution than the best feasible solution found using CPLEX. (Note some solutions are represented by the same point.) On each scenario and time step, our heuristic provides solutions at least as good as the best feasible solution found using CPLEX, and on several occasions, provides better solutions when CPLEX is unable to find an optimal solution in the given amount of processing time. In over two weeks of run time, we are unable to obtain a solution using CPLEX for the MEF scenario, so we are unable to determine the goodness of the

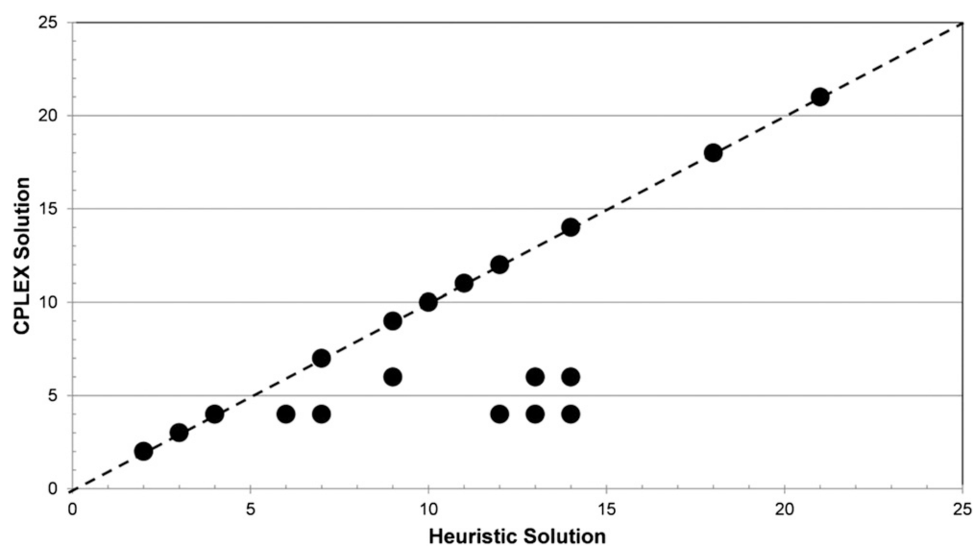

Figure 3. Comparison of the value of solutions obtained using CPLEX and the described heuristic. Each point represents a problem solved by both methods; a point below the diagonal axis indicates that the heuristic finds a better solution (i.e., fewer number of channels required) than the best feasible solution found by CPLEX under time constraints.

heuristically-obtained solutions. This is due to the vast size of the problem and the range of input values, and likely our own deficiencies in best utilizing CPLEX. See MCCDC (2013) for complete details on the size and runtimes of each scenario and time step.

We use the results of our optimization as the actual spectrum requirements for each of the scenarios at each time step. We provide detailed results in the next section.

ANALYSIS AND RESULTS

We instantiate each of our three scenarios in STK (Analytical Graphics, 2013) to calculate received signal strength ρ_{rs} along all wireless arcs $(r, s) \in W$, considering the effects of radio placement on terrain and various radio propagation phenomena. We use VBA to postprocess the outputs of STK, and then conduct optimization using the CPLEX optimizer (IBM, 2013a) and our heuristic.

Our scenarios dictate the number of units and the number and relative position of their assigned radios. We consider three tactical MAGTF scenarios, each with different network topologies. The first scenario involves a Marine Expeditionary Unit (MEU), comprising roughly 2,000 Marines, conducting an amphibious assault on an island. The second scenario is a Marine Expeditionary Brigade (MEB), comprising roughly 15,000 Marines, conducting irregular warfare (IW) operations in a desert environment. Our final scenario is a Marine Expeditionary Force (MEF), comprising roughly 60,000 Marines, conducting a major amphibious assault. We consider two different time steps for the MEU and MEB scenarios, and one for the MEF. For each scenario, we consider two different cases. In the first base case, wideband radios are provided to each echelon in a tactical organization from the battalion commander to the squad leader (or equivalent). In the reduced connectivity case, we provide wideband radios only from the battalion commander to the platoon commander (i.e., one level above the squad leader), in order to examine the corresponding reduction in spectrum requirements. A summary of these scenarios and their associated number of units and radios is displayed in Table 1. See MCCDC (2013) for further details on our scenarios.

In the following subsections, we present for each scenario the unconstrained channel requirements (i.e., the number of channels required by each MAGTF if there were no spectrum limitations) and the number of allocated channels (i.e., a realistic estimate of channel availability, provided by a qualified spectrum officer based on knowledge of the scenario, the corresponding EM environment, and his existing analytic tools). We compare these values with the best (i.e., lowest) solution found using either the exact optimization or heuristic methods, for both the base and reduced connectivity cases. As a form of sensitivity analysis, we vary the minimum required SIR values (which change the *max_interference* input values) from 10 to 40 dB. Ten dB is an extremely low SIR and data communications will likely be marginal or nonexistent at this level; 30 dB is a safe planning factor for high-data rate communications.

MEU Amphibious Assault Scenario. The MEU scenario depicts an amphibious assault to seize an island with desert terrain, fighting positions, and bunkers. Three reinforced rifle companies land on the island, one via surface vessels, and two via vertical insertion. The scenario challenges communications connectivity due to the

Table 1. Size of scenarios by number of Marines, units, and radios.

Scenario	Marines	Base case		Reduced connectivity	
		Units	Radios	Units	Radios
MEU time step 1	2,000	4	78	2	54
MEU time step 2	2,000	6	131	4	95
MEB time step 1	15,000	24	641	24	488
MEB time step 2	15,000	24	641	24	488
MEF	60,000	118	1,887	82	1,349

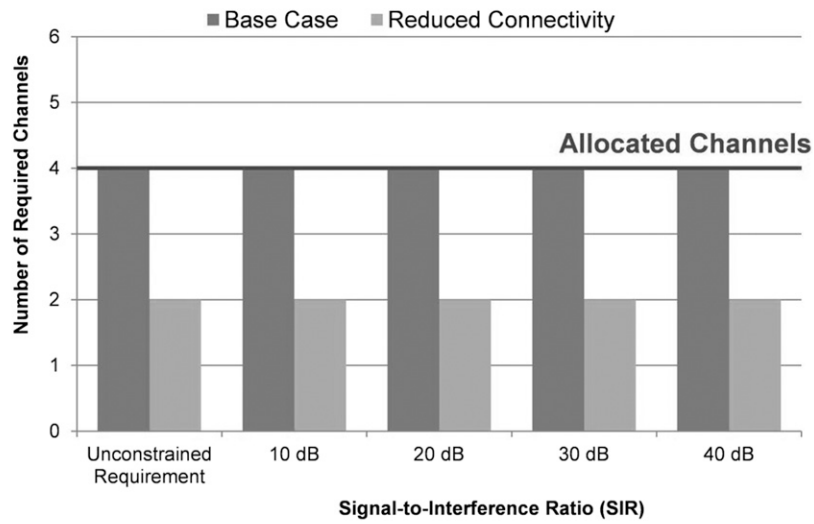

Figure 4. Number of required channels during the first time step of the MEU amphibious assault scenario. At each SIR level, there is no spectrum shortfall.

25 nautical mile distance between the ships (from which the landing force disembarks) and the landing zones. The first time step occurs while the amphibious assault is underway; the second occurs when most of the assault force is ashore. The allocated channel assignment for this scenario is four.

Figure 4 shows the allocated channel assignment and the number of required channels under the base case (blue bars) and reduced connectivity case (green bars). The unconstrained (i.e., no channel reuse) channel requirements appear on the left. Here, in the absence of channel reuse, the base case requires four channels and the reduced connectivity case requires two. For each SIR value (10 to 40 dB), the bars in Figure 4 show the number of channels required (calculated via our optimization methods) to support the base case and reduced connectivity cases at the indicated minimum SIR level. At each SIR level and for each case, channel reuse is not possible, i.e., the number of required channels determined

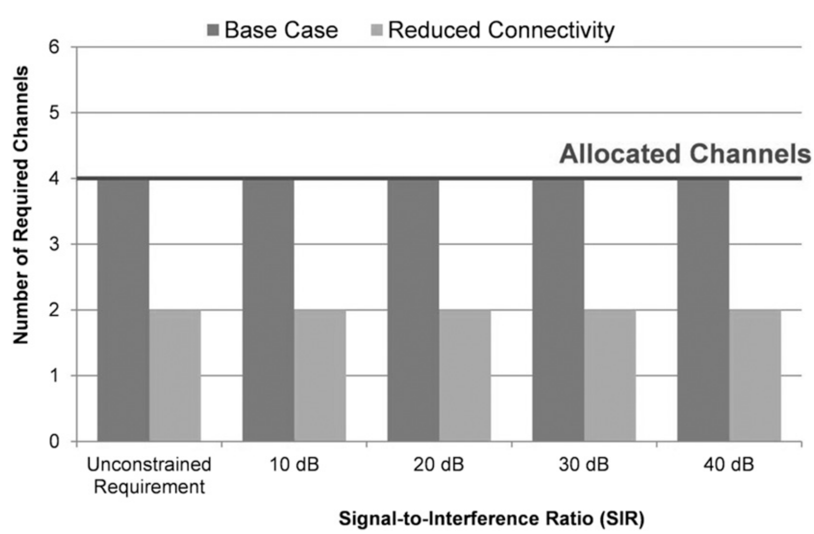

Figure 5. Number of required channels during the second time step of the MEU amphibious assault scenario. At each SIR level, there is no spectrum shortfall.

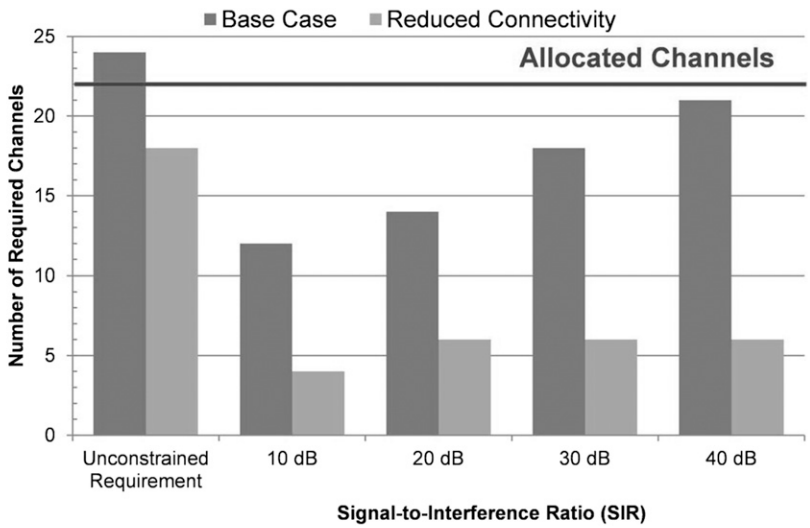

Figure 6. Number of required channels during the first time step of the MEB irregular warfare scenario. At each SIR level, there is no spectrum shortfall.

using optimization is never less than the unconstrained requirement. The number of required channels is always at or under the number allocated (i.e., the red line), so there is no spectrum shortfall in this scenario at this time step.

During the second time step, all assaulting units are ashore. Figure 5 shows the allocated channel assignment and the number of required channels. Channel reuse is possible and required to keep the number of required channels below the allocated channel assignment of four for the base case. Channel reuse is possible but not required for the reduced connectivity case. Overall, there is no spectrum shortfall in this scenario.

MEB Irregular Warfare Scenario. The MEB scenario depicts an irregular warfare conflict in relatively flat, desert terrain. The scenario considers three infantry battalions and a motorized battalion.

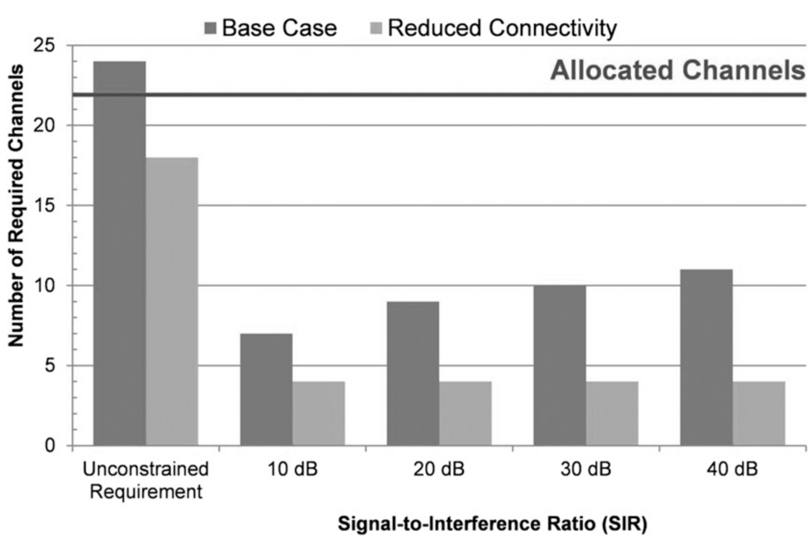

Figure 7. Number of required channels during the second time step of the MEB irregular warfare scenario. At each SIR level, there is no spectrum shortfall.

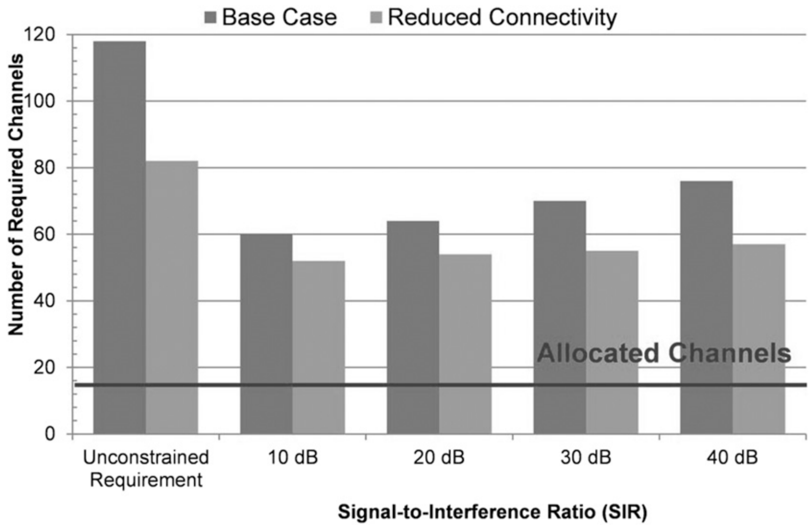

Figure 8. Number of required channels during the MEF amphibious assault scenario. At every SIR level, there is significant spectrum shortfall.

Each are operating from dispersed bases and send out patrols into the surrounding area. The scenario challenges line-of-sight communications due to the dispersion of forces. While the number of radios is much higher in this scenario than in the MEU scenario, the units are spread across a much larger area of operations. Two time steps are modeled in this scenario: Time Step 1 occurs while most of the forces are in their respective bases; Time Step 2 occurs when most of the forces are dispersed, approximating the situation when forces are on patrol. The allocated channel assignment for this scenario is 22.

Figure 6 shows that at the first time step, channel reuse provides a significant reduction in the number of required channels at all SIR levels. This keeps the optimized requirement below the allocated channel limit. For example, at 10 dB SIR, only 12 channels are required to support the base case, instead of 24 channels without channel reuse.

In the second time step of the MEB scenario, units have left their bases and are on patrol or are otherwise dispersed. Figure 7 shows the allocated channels and the number of required channels for the second time step. Channel reuse

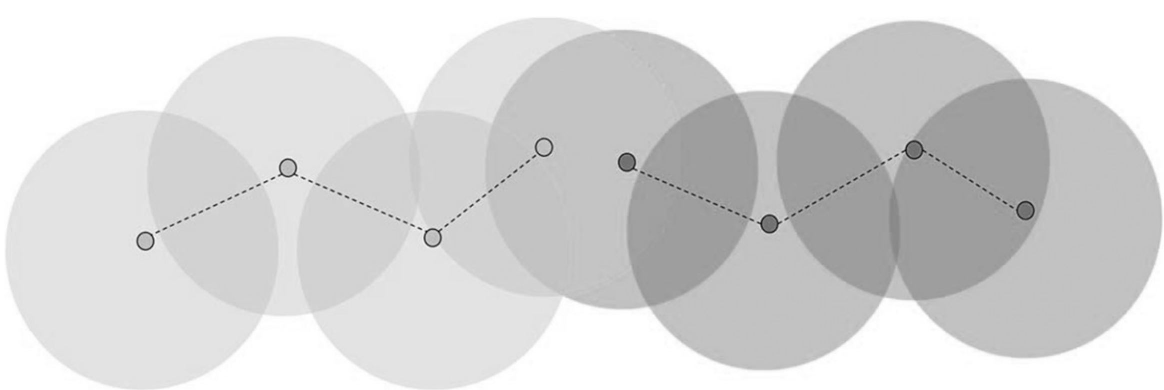

Figure 9. Depiction of two units with relatively high intraunit dispersion (i.e., large distances between radios in the same unit) and low interunit dispersion (i.e., small distances between units). This increases the strength of co-channel interference (i.e., the overlapping colors).

again provides a significant reduction in the number of required channels, and keeps the optimized requirement below the allocated channel assignment. Overall, there is no spectrum shortfall in this scenario.

MEF Amphibious Assault Scenario. The MEF scenario depicts a large amphibious assault, including 12 battalion landing teams and many other supporting units. Because of the vast complexity and long runtimes (i.e., greater than one week) associated with simulating this scenario, we consider only one time step. Specifically, we consider a moment when essentially all landing forces are ashore.

The number of allocated channels in this scenario is 14. It may seem counterintuitive that the larger MEF scenario has fewer allocated channels than the smaller MEB scenario. However, the context of each scenario is important. In the MEB scenario, forces are deployed in a desolate rural environment with few other joint or coalition units present. This allows a spectrum manager to allocate a relatively large portion of the spectrum to the MEB. In the MEF scenario, forces are participating as part of a very large joint and coalition operation, in a relatively noisy EM environment. There are more total demands on the spectrum yet the total spectrum at any location is always fixed; hence the amount of spectrum each force receives will be less. This tension is important enough to emphasize: If spectrum is assigned equitably, greater total spectrum demands imply each assignee receives less.

Figure 8 shows the allocated channels and the number of required channels for the MEF scenario. Channel reuse is possible, but does not get the requirement below the number of allocated channels. Overall, there is significant spectrum shortfall in the MEF scenario, regardless of whether connectivity is extended to the squad leader level or just the platoon commander level.

Summary of Results

We summarize our results for each scenario, time step, and case in Table 2. Neither optimization method (CPLEX or the heuristic) is able to reduce the channel requirement in the first time step of the MEU scenario. In all other scenarios and time steps, the heuristic and/or CPLEX is able to significantly reduce the total channel requirement.

Interunit and Intraunit Dispersion

We observe an interesting tension between interunit dispersion (i.e., the geographic proximity of disparate units) and intraunit dispersion (i.e., the geographic proximity of radios assigned to a particular unit). In the first time step of the MEU amphibious assault scenario, radios assigned to the same unit are relatively dispersed, i.e., units have high intraunit dispersion. This reduces the signal strength between radios on the same channel. At the same time, units are relatively close together, i.e., they have low interunit dispersion. This

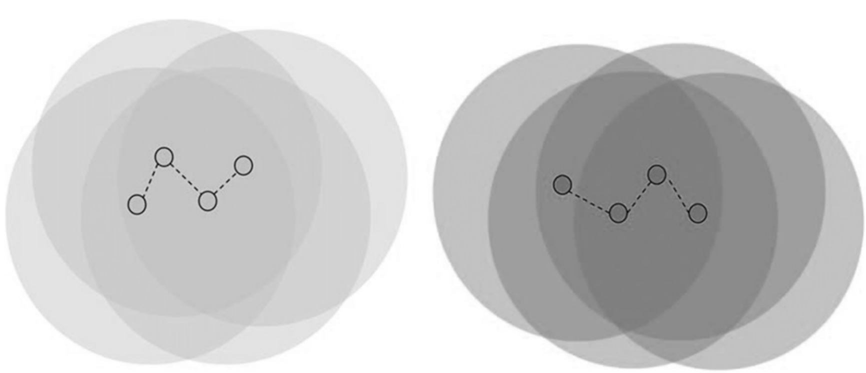

Figure 10. Depiction of two units with relatively low intra-unit dispersion (i.e., small distances between radios in the same unit) and high inter-unit dispersion (i.e., large distances between units). This reduces the strength of co-channel interference.

Table 2. Percentage reduction in channel requirements for each scenario, time step, and case, and the associated solution method (where ''both'' indicates both CPLEX and our heuristic provided the same solution).

Scenario	Case	Original	Solution	Reduction (%)	Solution method
MEU time step 1	Base	4	4	—	Both
	Reduced connectivity	2	2	—	Both
MEU time step 2	Base	6	2	67%	Both
	Reduced connectivity	4	2	50%	Both
MEB time step 1	Base	24	12	50%	Both
	Reduced connectivity	18	4	78%	Heuristic
MEB time step 2	Base	24	7	71%	Both
	Reduced connectivity	18	4	78%	Heuristic
MEF	Base	118	60	49%	Heuristic
	Reduced connectivity	82	52	37%	Heuristic

increases the strength of interference received by units operating on the same channel. This is depicted graphically in Figure 9. The combination of high intraunit dispersion and low interunit dispersion makes radios more susceptible to co-channel interference (i.e., the overlapping colors in Figure 9), and thus reduces the ability to reuse channels.

Conversely, in the second time step of the MEU scenario, units have low intraunit dispersion (increasing received signal strength among radios in the same unit) and high interunit dispersion (reducing the interference received by units operating on the same channel). This is depicted in Figure 10. The combination of low intraunit dispersion and high interunit dispersion makes radios less susceptible to co-channel interference, and thus increases the ability to reuse channels.

CONCLUSION AND RECOMMENDATIONS FOR FUTURE WORK

In nearly all of the scenarios and time steps we consider, spectrum reuse is required to enable the use of tactical wideband radios. The ability to reuse spectrum is largely determined by co-channel interference. This interference—coupled with the exponentially increasing number of solutions as the number of units increases—makes the channel assignment problem particularly difficult to solve. As demonstrated, it is infeasible for spectrum mangers to solve anything other than trivially small problems. Automation is required to determine efficient spectrum reuse; as demonstrated, our heuristic can quickly provide good solutions.

It must be emphasized that our analysis does not consider several important real-world phenomena that will affect the results, including harmonic interference between adjacent channels or interference from other radiation sources, which would further reduce the ability to reuse channels. Hence, our results should be viewed as the ''best case possible.'' In other words, in the real world it is quite likely that more channels will be required than indicated by our analysis, yet actual spectrum availability will remain the same or be reduced.

The most significant finding of this research is that even with optimal spectrum reuse and reduced network connectivity, there is insufficient spectrum available to support the full wideband spectrum requirement in the MEF scenario. Without sufficient spectrum, the number of radios must be reduced, or lower performance levels must be accepted. This problem may be even more significant for the US Army's Nett Warrior system, which will field MANET-capable smartphones to additional echelons of leadership (i.e., the fire team leader and perhaps even the individual soldier) (Office of the Secretary of Defense, 2013; Boland, 2013). The spectrum requirement can be further reduced through the use of cognitive radios, smart (or adaptive array) antennae, and dynamic network and spectrum assignment. However, the procurement and use of such systems is costly. We have presented the findings of this research to Marine Corps and Army leadership. We recommend an examination of the effectiveness and cost of employing technical solutions to reduce the overall MAGTF wideband spectrum requirement, and reconsideration of the radio fielding plan.

Although our heuristic provides solutions very quickly, a method of bounding the problem and thus gauging the goodness of each solution is highly desirable. Use of Lagrangian relaxation and dual relaxation methods may be able to provide such a bound (Nemhauser and Wolsey, 1999, pp. 323–331). Further investigation of our heuristic, as well as the use of other, nongreedy heuristic approaches (such as local search and genetic algorithms) are also certainly worth investigating. Additionally, more research is needed to investigate how to better handle the notoriously difficult cumulative interference constraints (M4) (see, e.g., Daniels et al., 2004; Palpant et al., 2008), which prevent us from exactly solving the larger problem instances.

This study does not consider the effects of other accidental interference or jamming. Although wideband, spread-spectrum, and frequency-hopping systems are in general more resistant to interference or jamming, even brute force noise jamming can be very effective and implemented with minimal cost and technical ability (Xu et al., 2004; Wood et al., 2007; *The Economist*, 2011). Such jammers will become increasingly prevalent in the future (Caro, 2007). Possible future research could include an examination of specific network disruptions (accidental or intentional) on the design and operation of MANETs (e.g., Grotschel et al., 1995, Shankar, 2008; Alderson et al., 2011; Nicholas and Alderson, 2015) and assigning spectrum dynamically to account for the highly mobile nature of the modern battlefield (Akyildiz et al., 2006; Zhao and Sadler, 2007).

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